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Leaving Cert Higher Level 2000 Paper 1 Solutions

Note the marks shown here are only a guideline and are only the opinion of the author !

Question 1

(a) $\frac{3x-5}{x-2}+\frac{1}{2-x}=\frac{3x-5}{x-2}-\frac{1}{x-2}=\frac{3x-6}{x-2}=3$ **(same question asked in 94) 10 marks**

(b) Standard proof see text Book . (20 marks)

(c) If $if\,(x-t)^2$ **is a factor of f(x) =** $x^3+3px+c$ **then x = t is a root of both f(x) = 0 and f'(x) = 0.**$\Rightarrow t^3+3tp+c=0$ **and** $3t^2+3p=0\Rightarrow p=-t^2$ **use this in the first equation to get** $c=2t^3$ **(10 marks each)**

Question 2

(a) Simultaneous equations Solution x = -1, y = 2, z = 2 .(10)

(b) Quadratic roots are x= 6 and x = -4

Just set $x+\frac{4}{x}=6,\Rightarrow x=3\pm\sqrt{5}..x+\frac{4}{x}=-4\Rightarrow x=-2,$ **they are still asking this type of question on the Junior cert (H) 20 marks (maybe!)**

(c) $(a^4-b^4)=(a^2-b^2)(a^2+b^2)=(a-b)(a+b)(a^2+b^2)$

$a^5-a^4b-ab^4+b^5=a^4(a-b)-b^4(a-b)=(a-b)(a^4-b^4)=(a-b)^2(a+b)(a^2+b^2)$

To show $a^5+b^5>a^4b+ab^4\Rightarrow a^5-a^4b-ab^4+b^5>0\Rightarrow(a-b)^2(a+b)(a^2+b^2)>0$

(20 marks)

Which is true since a,b >0 and a is not equal to b .

This is the third year in a row they have asked this question!

Question 3

(a) $B^{-1}=\begin{pmatrix}2........ & 1\\ -5.. & -3\end{pmatrix}$ $B^{-1}A=\begin{pmatrix}4...-1\\ -11...1\end{pmatrix}$ **(10)**

(b)(i) $\frac{-2+3i}{3+2i}=i,i^9=i$ **(ii)** $a=\pm4,b=\pm1$ **(20)**

(c) (i)Use a combination of DeMoivre and the Binomial to get the result wanted.
(ii)

$(-\sqrt{3-i}) = 2(Cos\frac{-5\pi}{6} + iSin\frac{-5\pi}{6}) \Rightarrow (-\sqrt{3}-i)^{10} = 2^{10}(Cos\frac{-25\pi}{3} + iSin\frac{-25\pi}{3}) =$ **(20)**

$2^{10}(\frac{1}{2} - i\frac{\sqrt{3}}{2}) = 2^9(1 - i\sqrt{3})$

Question 4

(a) Use $\frac{U_2}{U_1} = \frac{U_3}{U_2}$ **to find x , x = 1 and x = 11 (10)**

(b) Just replace n by n + 1 this gives

$1/2\{4^{n+1} - 2^{n+1}) = 1/2\{44^n - 22^n) = 24^n - 2^n = 4^n - 2^n + 4^n = 2(U_n) + 4^n$ **(nice) (20)**

(c) (1)This the first time since 1975 that an AP/GP has been asked ! It was not that obvious that they wanted the sum to infinity!

$g(x) = 1 + 2x + 3x^2 + 4x^3........nx^{n-1}......$

$x(gx)) = x + 2x^2 + 3x^3 + 4x^4$

$g(x)(1-x) = 1 + x + x^2 + x^3 + ... + x^{n-1} - nx^n$ **(10)**

$g(x)(1-x) = 1\frac{1-x^n}{1-x} - nx^n \Rightarrow g(x) = \frac{1-x^n}{(1-x)^2} - \frac{nx^n}{1-x}, n \to \infty g(x) = \frac{1}{(1-x)^2}$

(c) (ii) This is an great old question asked before in the early 1990's 1993 I think !(a CO'K Special) revived by ND

$P(n) = a.ar.ar^2.ar^3........ar^{n-1} = a^n(r.r^2.r^3.r^4.r \;\; .....r^{n-1}) = a^n(r^{1+2+3+4+.......n-1}) = a^n r^{(\frac{n-1}{2}(n+1))}$

(10) the powers of r is an AP .

Question 5

(a)$1.\dot{2} = \frac{11}{9}$ **(10)**

(b)The important bit is to show

$(k+1)! > 2^{k+1} \Rightarrow (k+1)(k!) > 22^k, ..(k+1) > 2.......k! > 2^k$ **therefore the statement is true (20).**

(c) (i) x =18. ©(ii) x = ln 2 , x = ln1 – ln 3 = - ln 3 (20)

Question 6

(a)(i) $3(1+5x)^2 \; (ii)\frac{(x-3)7 - 7x(1)}{(x-3)^2}$ **10 marks**

(b)Standard proof see text book (10 marks)

(b)(ii) $dy/dx = \frac{1}{\sqrt{1-(2x-1)^2}}.2.@.x = 1/2, dy/dx = 2$ **(10 marks)**

(c) (i)The vertical Asymptote is x = - 1 the horizontal Asymptote is y = 0
(ii)

$$dy/dx = \frac{-1}{(x+1)^2} \neq 0 \Rightarrow no..\max..no..\min, \frac{d^2y}{dx^2} = \frac{2}{(x+1)^3} \neq 0 \Rightarrow no..pt..of..\inf lection$$

dy/dx is equal at $x_1, x_2 \Rightarrow \frac{-1}{(x_1+1)^2} = \frac{-1}{(x_2+1)} \Rightarrow x_1^2 + 2x_1 + 1 = x_2^2 + 2x_2 + 1$ **just simplify to get** $x_1 + x_2 + 2 = 0$ **marks (10,5,5)**

this tpe of question was asked before!

Question 7

(a)dy/dx= 2 , the tangent is 2x-y-2=0. (10marks)

(b)
$$dx/dt = 3Cos^2t(-S\,\text{int}),..dy/dt = 3Sin^2t(Cost) \Rightarrow (dx/dt)^2 + (dy/dt)^2 =$$
$$(-3Cos^2tS\,\text{int})^2 + (3S\,\text{int}^2\,Cost)^2 = 9(CostS\,\text{int})\ \{Cos^2t + Sin^2t\} = \frac{9}{4}Sin^2 2t$$ **(20)**

(c)

$$f(x) = \frac{\ln x}{x} \Rightarrow dy/dx = \frac{x.\frac{1}{x} - \ln x.1}{x^2} = \frac{1-\ln x}{x^2} @..a\max..dy/dx = 0 \Rightarrow 1 - \ln x = 0 \Rightarrow \ln x = 1 \Rightarrow x = e, y = \tfrac{1}{e}$$

To show $x^e \leq e^x \Rightarrow e\ln x \leq x \Rightarrow \frac{\ln x}{x} \leq \frac{1}{e}$ **this is true because the maximum value of** $\frac{\ln x}{x}..is\,\frac{1}{e}$ **(20)**

Question (8)

(a)

$$\int (x^2+2)dx = \frac{x^3}{3} + 2x + c,..\int e^{3x}dx = \frac{e^{3x}}{3} + c,..(10)$$

$$(b).\int_0^{\frac{\pi}{2}} Sin3\Theta d\Theta = 1/2\int_0^{2\pi} 1 - Cos6\Theta d\Theta = \pi/4..(ii)\int_0^1 \frac{x}{x^2+4}dx, u = x^2 + 4 \Rightarrow du = 2xdx \Rightarrow du/2 = xdx$$

$$1/2\int_4^5 \frac{1}{u}du = 1/2\{\ln 5 - \ln 4\}$$

(10) each

(C) (i)

$$\int_2^p \frac{dx}{x^2 - 4x + 5} = \int_2^p \frac{dx}{1 + (x-2)^2} = Tan^{-1}(x-2)\Big|_2^p = Tan^{-1}(p-2) - Tan^{-1}(0) = \pi/4 \Rightarrow p - 2 = 1 \Rightarrow p = 3$$

(ii) $\int_0^k \sqrt{y}dy = \int_k^4 \sqrt{y}dy \Rightarrow \frac{2}{3}k^{\frac{3}{2}} = \frac{2}{3}[4^{\frac{3}{2}} - k^{\frac{3}{2}}] \Rightarrow 2k^{\frac{3}{2}} = 8 \Rightarrow k^{\frac{3}{2}} = 4 \Rightarrow k^3 = 16 \Rightarrow k = \sqrt[3]{16}$

(10)

Generally not a bad paper , most students if they put in the work should get a C it's nice to see the examiners delving into the archives of the old leaving cert for questions !