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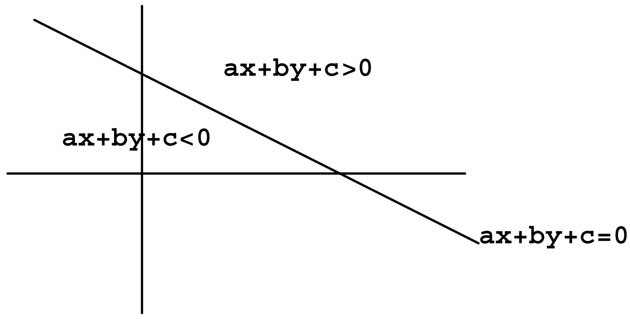
Notes on Coordinate Geometry

Sides of lines : The Line $ax + by + c = 0$ divides the coordinate plane into three areas :
(1)$ax + by + c > 0$, (2)$ax + by + c = 0$.(3)$ax + by + c < 0$.



If two points are on the <u>same side of a line</u> when the points are substituted in the line the answers have the <u>same sign</u>
Example 1 Investigate if (2,3) and (4,-2) are on the same side of the line 3x + 5y - 7= 0.
Sub in (2,3) to get 3(2) +5(3) - 7 = 14 > 0 .
Sub in (4,-1) to get 3(4) + 5(-2) - 7 = -5 < 0.
The results have opposite signs therefore the points are on opposite sides of 3x + 5y -7 = 0 .

The **formula $\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$ gives the perpendicular distance from the point (x_1, y_1) to the Line $ax + by + c = 0$.**
The formula can be used to find the following
(1) The distance from a point to a line :
Ex find the distance from (2,3) to the line 3x + 4y-8=0

$$\frac{3(2) + 4(3) - 8}{\sqrt{3^2 + 4^2}} = \frac{10}{5} = 2$$

(2)The dstance between two parallel lines
Ex Find the distance between the lines 5x + 12y +10 =0 and 5x +12y - 8 = 0 . Method find the distance between a point on one of the lines to the other line . Let y = 0 in the line 5x + 12 y + 10 = 0 . this gives x = -2 we now have the point (-2,0) on the line 5x + 12y + 10 = 0 .Use the formula to find the distance from (-2,0) to 5x + 12y -8 = 0 .

$$\frac{5(-2) + 12(0) - 8}{\sqrt{5^2 + 12^2}} = \left[\frac{-18}{13}\right] = \frac{18}{13}$$

(3)Find the equations of the lines which are 5 units on either side of 4x + 3y -12 = 0 .
The lines are of the form 4x + 3y + k = 0 . Find a point on 4x + 3y -12 = 0 (let x = 0 and find y) this gives (0,4) The distance from (0,4) to 4x + 3y + k = 0 is 5

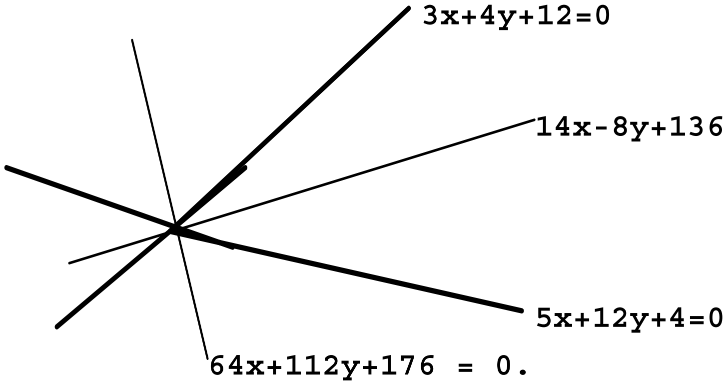
$$.\left|\frac{4(0) + 3(4) + k}{\sqrt{4^2 + 3^2}}\right| = 5. \Rightarrow \left|\frac{12 + k}{5}\right| = 5 \Rightarrow 12 + k = \pm 25 \Rightarrow k = -37, k = 13.$$

This gives the two Lines 4x+3y-37 = 0 . and 4x +3y + 13 = 0 .
Ex 4
Find the locus of the point such that it's distance from the line 3x + 4y + 12 = 0 is equal to it's distance from 5x + 12y + 4 = 0 .

Let (x,y) be the point then

$$\left|\frac{3x+4y+12}{\sqrt{3^2+4^2}}\right| = \left|\frac{5x+12y+4}{\sqrt{5^2+12^2}}\right| \Rightarrow 13(3x+4y+12) = \pm 5(5x+12y+4) \Rightarrow 39x+52y+156 = \pm(25x+60y+20)$$

This gives the two lines 14x - 8y+136 = 0 and 64x +112y +176 = 0 Notice these two lines are perpendicular and are in fact the equations of the bisectors of the angles between the two line3x+4y+12 = 0 and 5x+12y + 4 = 0



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