Leaving Cert Higher Paper 2.   2000

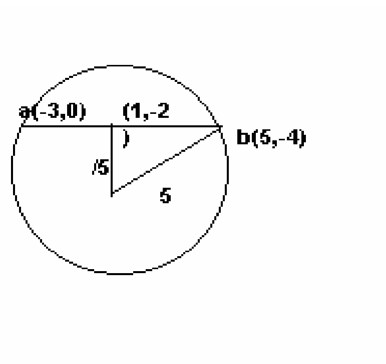
The marking scheme indicated is not the official marking scheme it is just the opinion of the author!

Question 1

(a) Slope of tangent is 7/9   (10)

(b)(i) The centre of the circle is (3, -2) the radius is 5
The centre of the second circle is (-6,10) its radius is $\sqrt{36+100-k}$,
The distance between the centres is 15 since the circles touch externally
$D = R_1 + R_2 \Rightarrow 15 = 5 + R_2 \Rightarrow 10 = \sqrt{136-k} \Rightarrow k = 36$ **(20)**

© The mid point of ab is (1, -2) we use Pythagoras to find the Radius of the circle, which is 5. We now know three bits of information.



$$\begin{aligned}
&C = x^2 + y^2 + 2gx + 2fy + c = 0 \\
&(a)(-3,0) \Rightarrow -6g + c = -3 \\
&(b)(5,-4) \Rightarrow 10g - 8f + c = -41 \\
&g^2 + f^2 - c = 25, \\
&\Rightarrow g = 0,\ f = 4, c = -9 \Rightarrow C_1 = x^2 + y^2 + 8y - 9 = 0 \\
&g = -2,\ f = 0, c = -21 \Rightarrow C_2 = x^2 + y^2 - 4x - 21 = 0
\end{aligned}$$

(20)

Question 2

(a) t=15,t=-15 (10)

$\vec{a}.\vec{b} = 0 \Rightarrow 2k^2 + (2k+3)(6) = 0 \Rightarrow 2k^2 + 12k + 18 = 0 \Rightarrow k^2 + 6k + 9 = 0 \Rightarrow k = -3$

(b) $\Rightarrow \overleftarrow{a} = 2\vec{i} - 3\vec{j}, \vec{b} = 9\vec{i} + 6\vec{j} \Rightarrow \vec{a} + \vec{b} = 11\vec{i} + 3\vec{j}$

$$Cos\Theta = \frac{22-9}{\sqrt{13}\sqrt{130}} = \frac{1}{\sqrt{10}} = 71.56^0 = 72^0$$

(20)

© $\vec{p}+\vec{q}=5\vec{i}-5\vec{j}$

$\vec{q}-\vec{p}=3\vec{i}+\vec{j}$

$\Rightarrow \vec{q}=4\vec{i}-2\vec{j} \Rightarrow \vec{p}=\vec{i}-3\vec{j} \Rightarrow$

$\vec{r}=\frac{10}{20}(4\vec{i}-2\vec{j})=2\vec{i}-\vec{j}$

$\vec{s}=\frac{7}{2}\vec{i}+m\vec{j}=k(2\vec{i}-\vec{j}) \Rightarrow 2k=\frac{7}{2} \Rightarrow k=\frac{7}{4}; m=-k \Rightarrow m=-\frac{7}{4}$ **(20)**

Question 3

(a) 3x- 7y – 23 = 0. (10)(maybe!)
(a)(1, -2). c (-4,8)
We know b divides [ac] in the ratio 3:2

$\vec{ab}=\frac{3}{5}\vec{ac} \Rightarrow \frac{3}{5}(-5,10)=(-3,6) \Rightarrow b=(-2,4)$

I have used translations here instead of the formula.

(b) $f = \begin{pmatrix} 2 & -3 \\ 6 .... & 1 \end{pmatrix} \begin{vmatrix} x \\ y \end{vmatrix} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ **f (a)=(8,4), f (b) = (-16, -8), f(c)=(-32, -16)**

$|f(a)f(b)|=\sqrt{720}, |f(b)f(c)|=\sqrt{320} \Rightarrow \frac{|f(a)f(b)|}{|f(b)f(c)|}=\frac{\sqrt{720}}{\sqrt{320}}=\frac{3}{2}$ **(20)**

©Area of Triangle rqu

$(0,0).(-2k+1,3k+5),(4,4) \Rightarrow \frac{1}{2}/(4(-2k+1)-4(3k+5)/=28 \Rightarrow k=2.$

$u=(-4,6), t=(x,y),..slope..ts=\frac{-3}{11},..slope..rs=1$

$\Rightarrow \frac{y-6}{x+4}=1,...slope...ts=\frac{y-9}{x-13}=\frac{-3}{11} \Rightarrow t(x=2, y=12)$

(20)

Question 4

(a) $A=\frac{1}{2}R^2\Theta=27 \Rightarrow \frac{1}{2}(36)\Theta=27 \Rightarrow \Theta=\frac{27}{18}=1.5 radians$ **(10)**

(b)

$15(1-Cos^2x)-4Cosx-11=0 \Rightarrow 15Cos^2x+4Cosx-4=0 \Rightarrow Cosx=.4 \Rightarrow x=66^0, x=-66^0$

$Cosx=-2/3 \Rightarrow x=132^0\ x=228^0$

(20)

©Standard Proof see text Book

Cos ((A+B) CosB+Sin (A+B) SinB (is in the form Cos (X-Y) where X =(A+B), Y=B
So we have Cos (A+B)-B)=CosA
(20)

Question 5

(a)(i) $\underset{x\to 0}{Lim}\frac{Sin5x}{x}=5..(ii)\underset{x\to 0}{Lim}\frac{Sin2x}{Sinx}=2$ **(10)**

(b) (i) Standard proof

(ii) $\frac{1-Tan^2(135-A)}{1+Tan^2(135-A)}=Cos(270-2A)=-Sin2A$ **(use page 9) (20)**

©(i)

$\frac{sr}{h}=Tan(45-B)\Rightarrow|sr|=h(Tan(45-B)$

$(ii)\frac{|qr|}{h}=Tan(45+B)\Rightarrow|qr|=h(Tan(45+B)\Rightarrow|qs|=h\{Tan(45+B)-Tan(45-B)\}$

$=h\{\frac{Sin(45+B)}{Cos(45+B)}-\frac{Sin(45-B)}{Cos(45-B)}=h\{\frac{Sin(45+B)Cos(45-B)-Cos(45+B)Sin(45-B)}{Cos(45+B)Cos(45-B)}$

$h\{\frac{Sin2B}{1/2Cos2B}\}=2hTan2B$

(20)

Question 6

(a)(i) 10,000, (ii) 9,000 (10)

(b) The characteristic equation is

$12x^2-8x+1=0\Rightarrow x=1/6,x=1/2\Rightarrow U_n=A(\tfrac{1}{2})^n+B(\tfrac{1}{6})^n$

$U_0=\frac{1}{15}\Rightarrow A+B=\frac{1}{15},U_1=\frac{7}{30}\Rightarrow A(\tfrac{1}{2})+B(\tfrac{1}{6})=\frac{7}{30}\Rightarrow A=\tfrac{2}{3},B=\tfrac{-3}{5}\Rightarrow U_n=\frac{2}{3}(\tfrac{1}{2})^n+\frac{-3}{5}(\tfrac{1}{6})^n$

$U_3=\frac{2}{3}(\tfrac{1}{2})^3+\frac{-3}{5}(\tfrac{1}{6})^3=\frac{29}{360}$

(20)

©(i) $\frac{6}{10}(\frac{5}{9})\times\frac{4}{10}(\frac{3}{9})$

(ii)($(\frac{3}{10}(\frac{2}{9})\times 2(\frac{4}{10})(\frac{3}{9})$

(iii) $\frac{3}{45}\times\frac{10}{45}=\frac{2}{135}$ **(20)**

Question (7)

(i) Number of Quadrilaterals is $C_4^6-6=9$ **(ii)** $C_2^4=6$ **(10)**

b

(i) $\{\frac{4}{52}\times\frac{4}{51}\times\frac{4}{50}\}\times 3!$ **(ii)** $\frac{4}{52}\times\frac{3}{51}\times\frac{2}{50}$

(iii) $\frac{26}{52}\times\frac{25}{51}\times\frac{13}{50}\times\frac{3!}{2!}$ **(iv)** $\frac{26}{52}\times\frac{25}{51}\times\frac{24}{50}+\frac{26}{52}\times\frac{25}{51}\times\frac{24}{50}$ **(20)**

c (i)

The mean and standard deviation are

$$\frac{\beta(q+r+s+t)}{4}+\frac{4\alpha}{4}=\beta\bar{x}+\alpha$$

$$\partial_x=\sqrt{\frac{(\beta(q-\bar{x}))^2+(\beta(r-\bar{x}))^2+(\beta(s-\bar{x})^2+(\beta(t-\bar{x}))^2}{4}}=\beta\delta_x$$ **(20)**

Question 8

(a)(i) $\lim_{n\to\infty}\frac{U_{n+1}}{U_n}=\lim_{n\to\infty}\frac{(n+3)!}{2^{n+3}}\times\frac{2^{n+2}}{(n+2)!}=\lim_{n\to\infty}\frac{n+3}{2}=\infty$ **The series diverges.**

(10)

(b)(i)

$$\int e^{2x}Cosxdx=\tfrac{1}{2}(e^{2x}Cosx+\tfrac{1}{2}\left[\tfrac{1}{2}e^{2x}Sinx-\int\tfrac{1}{2}e^{2x}Cosxdx\right]\Rightarrow$$

$$\tfrac{5}{4}\{\int e^{2x}Cosxdx\}=\tfrac{1}{2}e^{2x}Cosx+\tfrac{1}{4}e^{2x}Sinx\Rightarrow\int e^{2x}Cosxdx=\tfrac{2}{5}e^{2x}Cosx+\tfrac{1}{5}e^{2x}Sinx$$

$$\int_0^{\frac{\pi}{2}}e^{2x}Cosxdx=\frac{1}{n}\{e^{\pi}-2)=\left|\tfrac{1}{5}e^{2x}(2Cosx+Sinx\right|_0^{\frac{\pi}{2}}=[\tfrac{1}{5}e^{\pi}(2(0))+1)]-\tfrac{1}{5}e^{0}[2\cos 0+\sin 0]$$

$$=\tfrac{1}{5}e^{\pi}-\tfrac{1}{5}(2)=\tfrac{1}{5}(e^{\pi}-2)$$

(20)

©

$$y=\sqrt{r^2-x^2}\Rightarrow A=2x(2)\sqrt{r^2-x^2}\Rightarrow\frac{dA}{dx}=4x(\tfrac{1}{2}(r^2-x^2)^{\frac{-1}{2}}(-2x)+(r^2-x^2)^{\frac{1}{2}}4=0\Rightarrow$$

$$-4x^2+4r^2-4x^2=0\Rightarrow 8x^2=4r^2\Rightarrow x=\frac{r}{\sqrt{2}}\Rightarrow y=\frac{r}{\sqrt{2}}\Rightarrow A=(\sqrt{2}r)^2=2r^2$$

(20)

Comment

The paper was more difficult than paper 1. In particular Questions 3 and 6 were far too long!

Note while every effort has been made to give the correct solutions, we are human!